

University of Saskatchewan  
Department of Physics and Engineering Physics  
EP 228.3

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Midterm Examination, March 2004

*ONE SMALL FORMULA SHEET ALLOWED*

**Time:** 60 minutes

**Instructions:**

There are six questions each worth a different amount of marks

The exam is out of 49.

Read each question carefully and *THINK* before you *ACT*.

- 1) Express each of the complex expressions in terms of  $z = Me^{i\phi}$ . Write down  $M$  and  $\phi$  as single valued real numbers. Show some work or you get no marks. See last point in the instructions. (12 marks)

i)  $z = -2 + i3$

ii)  $z = \frac{de^{-iz_1}}{dt}$  where  $z_1 = \frac{\pi}{2}t - i0.3x$  evaluated at  $t = 0$  and  $x = 1$ .

iii)  $z = \left(9 + 4e^{-i\frac{\pi}{3}} - 4\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)7e^{i\frac{\pi}{4}}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

- 2) What are the first three terms in the MacLaurin Series expansion of  $e^{-x} \cos(2x)$ ? Remember the MacLaurin expansion is the Taylor Expansion about zero (6 marks)

- 3) Consider  $z_1 = A_1 e^{i\phi_1}$ ,  $z_2 = A_2 e^{i\phi_2}$  and  $z_3 = A_3 e^{i\phi_3}$  where  $A_1 = 2$ ,  $A_2 = \frac{1}{2}$ ,  $A_3 = \frac{1}{4}$ ,  $\phi_1 = \frac{2\pi}{3}t + \frac{\pi}{4}$ ,  $\phi_2 = \frac{5\pi}{3}t$  and  $\phi_3 = \frac{4\pi}{3}t$ . What is the derivative of the function that describes the real part of  $z_1 + z_2 + z_3$ ? (5 marks)

- 4) Show, using the complex representation of  $\cos \theta$  and  $\sin \theta$ , that: (8 marks)

$$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$$

5) Prove  $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$  using the principles of induction and the basis case of  $n = 1$ . (8 marks)

6) Given the set of data points :

$(-2, 12), (-1, 7), (0, 4), (1, 3)$  and  $(2, 4)$

that were measured, without error, by sampling a phenomena that can be described by a perfect parabola, i.e. :  $f(t) = a_2 t^2 + a_1 t + a_0$

- Determine the numerical integral of the sampled curve using 0<sup>th</sup>, 1<sup>st</sup> and 2<sup>nd</sup> order polynomial approximations. **IMPORTANT:** There is a mistake to be made on this particular question that will result in the forfeiture of all 4 marks. (4 marks)
- Determine the central difference numerical derivative at the applicable points. (2 marks)
- What is the exact value of the integral? (2 marks)
- What are three equations required to determine the coefficients,  $a_2$ ,  $a_1$ , and  $a_0$ , for the exact quadratic fit? DO NOT SOLVE JUST WRITE DOWN THE EQUATIONS!!! (2 marks)

End of Exam!

$$\begin{aligned}
 a_n &= a_{n-1} + n(n+1) = \frac{n(n+1)(n+2)}{3} \\
 a_{n+1} &= a_n + (n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3} \\
 \frac{(n+1)(n+2)(n+3)}{3} &= a_{n-1} + n(n+1) + (n+1)(n+2) \\
 \frac{(n+1)(n+2)(n+3)}{3} &= \frac{n(n+1)(n+2)}{3} - n(n+1) + n(n+1) + (n+1)(n+2) \\
 \frac{(n+1)(n+2)(n+3)}{3} &= (n+1)(n+2) + \frac{n(n+1)(n+2)}{3} \\
 (n+1)(n+2)(n+3) &= 3(n+1)(n+2) + n(n+1)(n+2) \\
 (n+1)(n+2)(n+3) &= (3+n)(n+1)(n+2) \quad \checkmark
 \end{aligned}$$